The hyperon neutron star mean field model

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Abstract. – In this paper the impact of the strength of hyperon-nucleon and hyperon-hyperon on the maximum neutron star mass is shown. The strong hyperon-hyperon coupling constant together with the additional nonlinear meson interaction terms lead to the essential softening of the equation of state and to the existence of another more compact branch of stable neutron star configurations. The analysis of the mass-radius relations leads to the conclusion that the additional stable configurations of hyperon-rich neutron stars are characterized by the reduced value of radii.

The theoretical framework employed in this paper is the relativistic nuclear mean field theory extended by allowing for additional non-linear meson couplings. The chiral effective Lagrangian proposed by Furnstahl, Serot and Tang (FST) [1], [2] constructed on the basis of the effective field theory and density functional theory for hadrons gave in the result the extension of the standard relativistic mean field theory and introduced additional non-linear scalar-vector and vector-vector self-interactions. This Lagrangian in general includes all non-renormalizable couplings consistent with the underlying symmetries of QCD. Applying the dimensional analysis of Georgi and Manohar [3], [4] and the concept of naturalness one can expand the nonlinear Lagrangian and organize it in increasing powers of the fields and their derivatives and truncated at given level of accuracy [5]. If the truncated Lagrangian includes terms up to the forth order it can be written in the following form

$$
\begin{aligned}
\mathcal{L} &= \sum_B \bar{\psi}_B(i\gamma^\mu D_\mu - m_B + g_{\sigma B}\sigma + g_{\sigma^* B}\sigma^*)\psi_B \qquad (1)\\
&+ \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2(\frac{1}{2} + \frac{\kappa_3}{3!}\frac{g_{\sigma B}\sigma}{M} + \frac{\kappa_4}{4!}\frac{g_{\sigma B}^2\sigma^2}{M^2}) + \frac{1}{2}\partial_\mu\sigma^*\partial^\mu\sigma^* - \frac{1}{2}m_{\sigma^*}^2\sigma^{*2}\\
&+ \frac{1}{2}m_\phi^2\phi_\mu\phi^\mu - \frac{1}{4}\phi_{\mu\nu}\phi^{\mu\nu} - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}(1+\eta_1\frac{g_{\sigma B}\sigma}{M} + \eta_2\frac{g_{\sigma B}^2\sigma^2}{2M^2})m_\omega^2\omega_\mu\omega^\mu\\
&- \frac{1}{4}R_{\mu\nu}^a R^{a\mu\nu} + (1+\eta_\rho\frac{g_{\sigma B}\sigma}{M})\frac{1}{2}m_\rho^2\rho_\mu^a\rho^{a\mu} + \frac{1}{24}\zeta_0 g_{\omega B}^2(\omega_\mu\omega^\mu)^2.
\end{aligned}
$$

$\Psi_B^T = (\psi_N, \psi_\Lambda, \psi_\Sigma, \psi_\Xi)$. The covariant derivative D_μ is defined as

$$D_\mu = \partial_\mu + ig_{\omega B}\omega_\mu + ig_{\phi B}\phi_\mu + ig_{\rho B}I_{3B}\tau^a\rho_\mu^a \qquad (2)$$

whereas $R_{\mu\nu}^a$, $\Omega_{\mu\nu}$ and $\phi_{\mu\nu}$ are the field tensors. m_B denotes baryon mass whereas m_i ($i = \sigma, \omega, \rho, \sigma^*, \phi$) are masses assigned to the meson fields, M is the nucleon mass. In the high

density regime in neutron star interiors when the Fermi energy of nucleons exceeds the hyperon masses hyperons are expected to emerged due to the strangeness changing interactions [6], [7], [8], [9], [10]. The appearance of these additional degrees of freedom and their impact on a neutron star structure have been the subject of extensive studies. In order to reproduce attractive hyperon-hyperon interaction two additional hidden-strangeness mesons, which do not couple to nucleons, have been introduced, namely the scalar meson $f_0(975)$ (denoted as σ^*) and the vector meson $\phi(1020)$ [11].

Due to the fact that the expectation value of the ρ meson field is an order of magnitude smaller than that of ω meson field, the Lagrangian function (2) does not include the quartic ρ meson term. In addition, as this paper deals with the problem of infinite nuclear matter the terms in the original Lagrangian function (see [1], [2]) involving tensor couplings and meson field gradients have been excluded. There are two parameter sets presented in the original paper by Furnstahl et al [1], [2] G1 and G2 which have been determined by calculating nuclear properties such as binding energies, charge distribution and spin-orbit splitting for a selected set of nuclei [12]. For the purposes of this paper two parameter sets have been chosen: the G2 parameter set and the extended TM1 parameter set denoted as TM1*. The latter one constructed by Del Estal et al [13] represents the standard TM1 parameter set supplemented with additional nonlinear couplings stemming from the effective field theory. Calculations performed with the TM1* parameter set properly reproduce properties of finite nuclei. Both parameter sets G2 and TM1* make it possible to compare the obtained results with the Dirac-Brueckner-Hartree-Fock (DBHF) calculations for nuclear and neutron matter above the saturation density. The DBHF method results in a rather soft equation of state in the vicinity of the saturation point and for higher densities. Calculations performed on the basis of the effective FST Lagrangian with the use of the G2 and TM1* parameter sets predict similar, soft equation of state.

The theoretical description of strange hadronic matter properties is given within the relativistic mean field approximation. In this approximation meson fields are separated into classical mean field values and quantum fluctuations which are not included in the ground state. In the case of a hyperon-rich matter the considered parameterization has to be supplemented by the parameter set related to the strength of the hyperon-nucleon and hyperon-hyperon interactions. The scalar meson coupling to hyperons can be calculated from the potential depth of a hyperon in the saturated nuclear matter. Assuming the SU(6) symmetry for the vector coupling and determining the scalar coupling constants from the potential depths, the hyperon-meson couplings can be fixed.

The strength of hyperon coupling to strange meson σ^* is restricted through the following relation [10] $U_{\Xi}^{(\Xi)} \approx U_{\Lambda}^{(\Xi)} \approx 2U_{\Xi}^{(\Lambda)} \approx 2U_{\Lambda}^{(\Lambda)}$. which together with the estimated value of hyperon potential depths in hyperon matter provides effective constraints on scalar coupling constants to the σ^* meson. The currently obtained value of the $U_{\Lambda}^{(\Lambda)}$ potential at the level of 5 MeV [14] permits the existence of additional parameter set [15] which reproduces this weaker $\Lambda\Lambda$ interaction. Throughout this paper this parameter set is referred to as weak, whereas strong denotes the stronger $\Lambda\Lambda$ interaction for $U_{\Lambda}^{(\Lambda)} \simeq 20$ MeV [11]. In the nucleon sector the saturation properties of nuclear matter namely the saturation density ρ_s, binding energy E_b, compressibility K and Dirac effective nucleon mass $m_{effN} = M - g_{\sigma N}\sigma$ determined for the given parameter sets. Neutron star matter is considered as a system with conserved baryon number $n_b = \sum_B n_B$ ($n_B = k_B^3/3\pi^2$ denotes the number density of species B, $B = n, p, \Lambda, \Xi^-, \Xi^0$) and electric charge being in chemical equilibrium with respect to weak decays. In general, weak processes for baryons can be written in the following form $B_1 + l \leftrightarrow B_2$ where B_1 and B_2 are baryons, l denotes lepton. Provided that the weak processes stated above take

Table I – *Chosen parameter sets.*

	m_σ	$g_{\sigma N}$	$g_{\omega N}$	$g_{\rho N}$	η_1	η_2	η_ρ	ζ_0	κ_3	κ_4
G2	520.25	10.496	12.76	9.48	0.65	0.11	0.39	2.64	3.247	0.63
TM1*	511.20	11.22	14.98	10.00	1.1	0.1	0.45	3.6	2.513	8.97

place in thermodynamic equilibrium the following relation between chemical potentials can be established $\mu_B = q_B\mu_n - q_{e_B}\mu_e$. This relation involves two independent chemical potentials μ_n and μ_e corresponding to baryon number and electric charge conservation, μ_B denotes chemical potential of baryon B with the baryon number q_B and the electric charge q_{e_B}. The baryon effective mass is defined as $m_{effB} = m_B - g_{\sigma B}\sigma_0 - g_{\sigma^* B}\sigma_0^*$. Thus the β equilibrium conditions for Λ, Σ and Ξ hyperons lead to the following relations:

$$\mu_\Lambda = \mu_{\Sigma^0} = \mu_{\Xi^0} = \mu_n, \qquad \mu_{\Sigma^-} = \mu_{\Xi_-} = \mu_n + \mu_e, \qquad \mu_p = \mu_{\Sigma^+} = \mu_n - \mu_e. \tag{3}$$

The conditions of charge neutrality and β-equilibrium imply the presence of leptons which are introduced as free particles. Muons start to appear in neutron star matter in the process $e^- \leftrightarrow \mu^-$ after μ_μ has reached the value equal to the muon mass. The appearance of muons not only reduces the number of electrons but also affects the proton fraction.
Solving the hydrostatic equilibrium equation of Tolman, Oppenheimer and Volkov, which allows one to construct theoretical model of a neutron star and to specify and analyze its properties, demands the specification of the equation of state. In Fig.1 the pressure of a neutron star matter as a function of the energy density for the TM1 and TM1* parameter sets has been presented. This figure includes results for the strong and weak $Y-Y$ interactions. For comparison the equations of state for the matter with zero strangeness has been included. Dashed curves represent the equations of state obtained for neutron star matter without hyperons. The parameter set TM1* leads to more soft equation of state than the TM1 one. This softening is caused by the presence of additional nonlinear couplings. In general the inclusion of hyperons softens considerably the equation of state at high densities. This effect is maximized in the case of strong $Y-Y$ interaction and by adding nonlinear terms. The form of the equation of state has profound consequences for the maximum mass of a neutron star. Solutions of the Oppenheimer-Tolman-Volkov equation for the considered equations of state are presented in Fig. 4. The obtained mass-radius relations are constructed for neutron star matter with hyperons and compared with the mass-radius relation for non-strange matter. The analysis has been done for the ordinary TM1 parameter set [16] and for TM1*. The TM1 parameter set gives larger masses than the TM1* one. However, the key difference between the TM1 and TM1$*$ mass-radius diagrams lies in the results obtained for the strong hyperon-

Table II – *Strange scalar sector parameters.*

		$g_{\sigma\Lambda}$	$g_{\sigma\Xi}$	$g_{\sigma^*\Lambda}$	$g_{\sigma^*\Xi}$
G2	weak	6.410	3.337	3.890	11.643
	strong	6.410	3.337	7.931	12.458
		$g_{\sigma\Lambda}$	$g_{\sigma\Xi}$	$g_{\sigma^*\Lambda}$	$g_{\sigma^*\Xi}$
TM*	weak	6.971	3.583	5.526	13.450
strong	6.971	3.583	9.069	14.394	

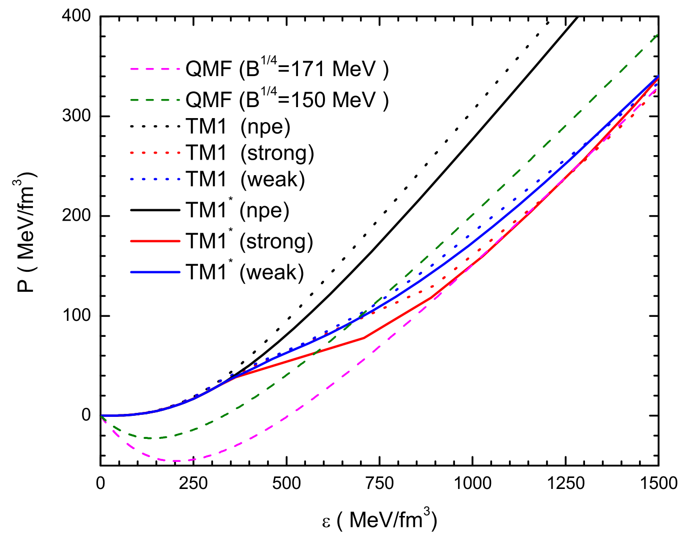

Figure 1 – (color online) The equation of state calculated for TM1 and TM1* parameter sets. Dotted curves represent the results obtained for TM1 parameter set. The influence of hyperons is considered for the weak and strong $Y-Y$ interactions. The case of non-strange baryonic matter is marked by npe. Dashed curves show the quark matter equations of state obtained for two fixed values of the bag parameter: $B^{1/4} = 150$ MeV and $B^{1/4} = 171$ MeV (B_{crit})

hyperon interaction. In this case for TM1* parameter set besides the ordinary stable neutron star branch there exists the additional stable branch of solutions which are characterized by a similar value of masses but with significantly reduced radii. For the purpose of this paper A denotes the maximum mass configuration of the ordinary neutron star branch whereas B the additional maximum. The comparison of the maximum mass configurations obtained for the weak and strong $Y-Y$ interactions makes it possible to estimate the role of the hyperon-hyperon couplings strength. The strong model gives the reduced value of the maximum mass. The reduction is of the order of 0.1-0.2 $M_{\odot}$. The influence of the strength of the hyperon-nucleon couplings has been analyzed for a very limited range of the $g_{\sigma\Lambda}$ parameter strictly connected with the value of the potential felt by a single Λ in saturated nuclear matter. The chosen values of the potential are: -27 MeV, -28 MeV, -30 MeV. This leads to the following $g_{\sigma\Lambda}$ parameters: 9.203, 9.158, 9.069. As it was stated, the additional nonlinear meson-meson interaction terms and the strong hyperon-hyperon interaction create necessary conditions for the existence of the second branch of stable neutron star configurations. For the ordinary neutron star configurations the well known results has been obtained i.e.the weaker the coupling $g_{\sigma\Lambda}$ the lower is the value of the maximum mass. However, for the additional branch the analysis of the mass radius relation Fig.4 gives the opposite result. Similar conclusions can be drown from Fig.2. In this figure the mass as a function of the central density is depicted. Particular curves are labelled by the three chosen values of the

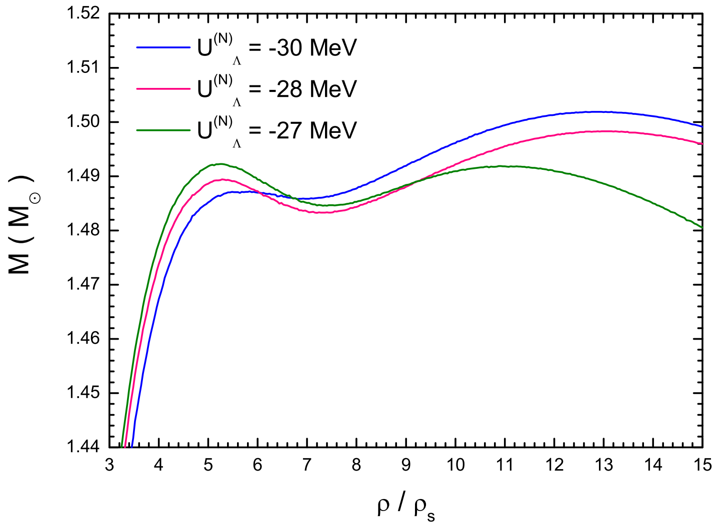

Figure 2 – (color online) The mass-central density relation. The left panel shows results for the ordinary TM1 parameter set whereas the right panel presents the mass-radius relation for the TM1* parameter set.

$U_\Lambda^{(N)}$ potential. The composition of hyperon star matter as well as the threshold density for hyperons are altered when the hyperon-hyperon interaction strength is changed. Fig. 3 presents fractions of particular baryon species Y_B as a function of the density ρ for the strong model of the TM1* parameterization. At very low density neutrons and protons are the most abundant baryons. The first strange baryon that emerges is Λ and is followed by Ξ^- and Ξ^0. The maximum mass configuration A resembles an ordinary neutron star predominantly composed of nucleons. Hyperons (Λ and Ξ^0) appear in the innermost part of the star. The appearance of the new branch on of stable solutions on the mass-radius diagram makes it possible to exist more dense stars with increased concentrations of hyperons. The appearance of Ξ^- hyperons at higher densities through the condition of charge neutrality affects the lepton fraction and causes a drop in their contents. Thus the presence of charged hyperons permits lower lepton content and in the case of the additional stable branch objects charge neutrality tends to be guaranteed without lepton contribution. The properties of strange neutron stars has been studied with the use of the improved TM1 parameter set which include additional nonlinear coupling stemming from the effective field theory. This parameter set in the strange sector has been supplemented by parameters that are related to the strength of the of hyperon-hyperon and hyperon-nucleon interactions. The impact of the strength of hyperon interactions on neutron star masses has been analyzed. This analysis shows that there exists very strong correlation between the value of the maximum neutron star mass and the strength of hyperon coupling constants. The inclusion of additional nonlinear meson interaction terms which modify the high density behavior of the equation of state together with the strong hyperon-

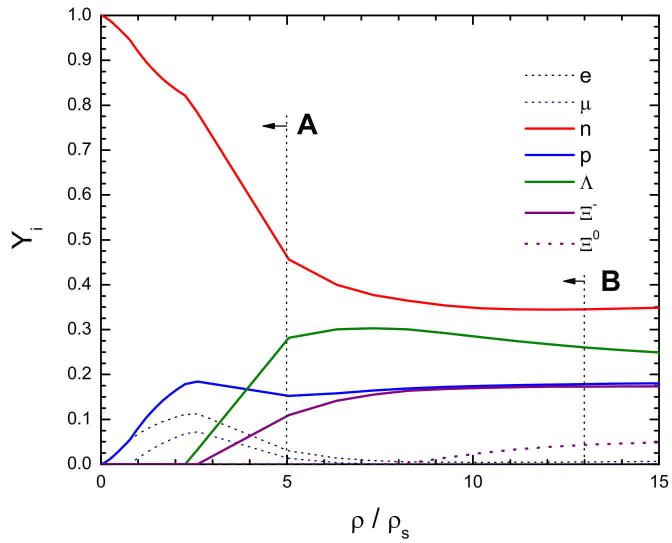

Figure 3 – (color online) Baryon and lepton concentrations in neutron star matter as a function of density ρ ($\varepsilon = \rho c^2$) where ε denotes the energy density.

hyperon interaction lead to the existence of additional stable stellar configurations with similar masses and smaller radii than an ordinary neutron star. The reduction in radius is of the order of 2.5 km. The internal composition of this additional neutron star configurations is almost completely free of leptons. Transmutation similar to the phase transition from the ordinary neutron star to the more compact hyperstar may be the main origin of the short gamma ray burnst [17].

In order to complete the analysis of the existence of the very compact, hyperon rich stars (hyperstar), the conditions under which the occurrence of quark matter and the formation of stable configurations of hybrid stars have to be established. Two phases of matter have been compared: the strange hadronic matter and quark matter. The phase with the highest pressure (lowest free energy) is favored. In Fig.1 the equation of state of the quark matter obtained with the use of the quark mean field model [19] with the direct coupling of the bag parameter to the scalar meson fields σ_0 and σ_0^*, is also included. The results strongly depend on the value of the bag parameter. There exists the limiting value of $B_{crit}^{1/4} = 171$ MeV, for $B > B_{crit}$ there is no quark phase in the interior of a neutron star. The bag parameters $B < B_{crit}$ lead to the intersection of the quark matter equation of state and that of hyperon star matter with the strong $Y - Y$ interactions. Thus a hybrid star with a quark phase inside can be constructed. The obtained mass-radius relation for hybrid stars involving quark matter is presented in the right panel of Fig.4 (for $B^{1/4} = 150MeV < B_{crit}^{1/4}$). The maximum hybrid star mass is of about $1.52M_\odot$ and is greater than the maximum mass of the very compact additional stable branch configurations (B). The radius of the hybrid star maximum mass configuration is also greater than that of the configuration marked as B.

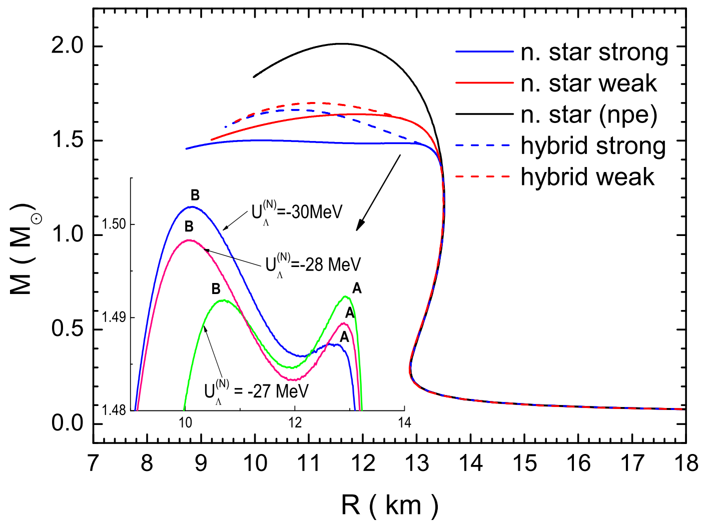

Figure 4 – (color online) The mass-radius relation for the TM1* parameter sets.

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